Modern Portfolio Theory vs Behavioral Finance:

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Modern Portfolio Theory (MPT):

The last century witnessed a rise in the use of mathematics in the field of finance, such as the expected value or the expected utility theories, in investment decisions. One of the protagonists of the Modern Portfolio Theory is Nobel Prize's winner Harry Markowitz, with his work on the mathematical framework for investments decisions known as Markowitz model.

Modern Portfolio Theory is generally based on a particular mathematical concept of preferences, the mean – variance stochastic dominance, whose goal is to construct portfolios looking at their expected values and their variance or, in terms of standard deviation, their volatility. The expected return of the portfolio is the average of the returns of the various assets weighted with the quotas is given by the formula:

$$E(P) = \sum_i \omega_i E\left(A_i\right)$$

Where $E(P)$ is the expected return of the portfolio, ωi are the quotas for each asset and $E(Ai)$ is the expected return of each asset.

Volatility is defined as the standard deviation of the portfolio return (σp):

$$\sigma_p = \sqrt{\sum_i \omega_i^2 \sigma_i^2 + \sum_i \sum_{i \neq j} \omega_i \omega_j \sigma_i \sigma_j \rho_{ij}}$$

Where σ_i and σ_j are the volatility of the assets i and j, and ρ are the Pearson's correlation coefficient, it describes the linear correlation between the return of the two assets.

The Markowitz model, assuming that we are working in an efficient market, implies that risk-averse individuals can aid in building an efficient portfolio and, and given their utility function $u(\cdot)$, we can derive an optimized portfolio which maximizes individual profit.

In the model the role of the efficient frontier is crucial; the efficient frontier it's mathematically defined as a hyperbolic function derived from the constrained minimization of the portfolio volatility (in other words, after choosing an expected return for our portfolio, we want to minimize its volatility). The analytic expression of the hyperbole is:

$$\sigma = \sqrt{\frac{cm^2 - 2bm + a}{ac - b^2}} \quad a = \mu^T V^{-1} \mu \quad b = e^T V^{-1} \mu \quad c = e^T V^{-1} e$$

Where e it's the column vector with unit components, μ it's the column vector of the rates of return average return of the assets present in the portfolio, V^{-1} it's the inverse matrix of the variance – covariance matrix of the assets and m it's the expected return of the portfolio.

The previous formula allows us to determine the efficient frontier and the efficient portfolio. By selecting a particular return for our portfolio (so selecting a value for m), we can input it in the formula and, also knowing the average returns and correlation coefficient of the various assets, we can calculate the minimum volatility value of the portfolio. To summarize, the efficient frontier it's a part of the hyperbole that contains an infinite number of efficient portfolios, defined by expected return and volatility. After calculating the frontier, we can begin to consider single individuals and their personal optimal portfolio. By using a utility function such as $u(t) = t - \frac{\alpha t^2}{2}$ we can calculate the optimal portfolio by maximizing the expected utility of the portfolios while staying on the efficient frontier. The formula that derives from this states that:

$$m^* = \frac{ac - b^2 + \alpha b}{\alpha(ac - b^2 + c)}$$

So, the optimal portfolio for an individual with a specific α has an expected return of m^*. For instance, let's suppose that an individual has an α = 1 and that the portfolio is composed of n assets so that a = 5, b = 84 and c = 1446. With these numbers the efficient frontier is:

$$\sigma = \sqrt{8.3m^2 - 0.97m + 0.029}$$

And the optimal expected return for the individual is:

$$m^* = \frac{(5\times1446 - 84^2 + 84)}{5\times1446 - 84^2 + 1446} = 0.159$$

So, an expected return of 16%, with the efficient frontier we can calculate the minimum volatility of this portfolio:

$$\sigma = \sqrt{8,3\times0,159^2 - 0,97\times0,159 + 0,029} = 0.292$$

In conclusion, for an individual with an α of 1 and with an efficient frontier like the one above, the optimal portfolio has an expected return of 16% with a volatility of 29,2%.

Modern Portfolio Theory is widely used in finance, and it is very useful for selecting efficient portfolios by diversifying investments, also allowing us to calculate the specific quotas to invest in each available asset, risky or not. Within the Modern Portfolio Theory, the contribution given by Fama, in the early years of 1960, is also extremely important, in fact, other crucial assumptions are made to make the model work, such as the rationality of the market agents and the "correctness" of the offered asset's prices. The importance of diversification as a tool for managing market risks and the fundamental role of correlation between assets is also underlined in order to try to understand market behavior. The model is highly effective thanks to its objectivity and direct simplicity, deriving from its purely mathematical nature, which leaves little room for interpretation. The model however it's not perfect. The Modern Portfolio Theory problems are evident, starting from the necessary basic assumption. In order for the model to work one must assume that the market is efficient, every individual is perfectly rational and can be described by a mathematical function. It is clear that all these assumptions are part of a simplified reality and they can't describe optimal strategies fully. We have to consider the role of the behavioral theory into our financial research to improve our market forecasting capabilities and to build truly efficient portfolios.

Behavioral Finance (BF):

While the Modern Portfolio Theory is a popular investment approach, empirical data indicates that investors frequently display cognitive biases that influence their investment choices and thus are not always rational. The idea that psychological factors can drive the price of a stock conflicts the premise that markets are efficient. The goal of behavioral finance (BF), a discipline that blends psychology and finance, is to comprehend these biases and how they affect the way people make financial decisions. This section of the article will explore how BF contradicts certain ideas of the MPT through cognitive biases.

Overconfidence Bias:

Overconfidence bias highlights the tendency for people to overestimate their skills or the accuracy of their beliefs. In the context of investing, overconfidence bias can cause investors to take excessive risks or make investment decisions that are too optimistic, because they think their knowledge of the market and their skills are superior to others.

According to Odean (1999), overconfident investors believe they are able to outsmart the market, leading them to trade more frequently. It has been demonstrated that such investors tend to underperform in the long run and aggressive trading often yields below-average investment returns due to transaction costs and faulty timing choices. Another study by Barber and Odean (2001) has demonstrated that overconfidence often leads to keeping more concentrated portfolios and investing in speculative equities, which entails more risk and worse returns.

The dot-com boom of the late 1990s perhaps is one of the biggest illustrations of the consequences of this bias. At the time, many investors overestimated the potential of internet and technology related firms and made considerable investments in them, only to incur significant losses when the bubble burst (Baker and Wurgler, 2007).

Loss Aversion Bias:

Loss aversion is the propensity to prefer the joy of gains to the pain of losses. Due to the emotional suffering realizing a loss brings about, investors are often hesitant to sell loss-making investments, even if doing so is the wiser course of action.

Tversky and Kahneman's (1991) research revealed that investors frequently cling onto losing stocks for an excessive amount of time in the hopes of recouping their losses while selling winning equities too soon to lock in gains. This disposition effect, a phenomenon where investors realize gains too early and hold on to losses for an excessive amount of time, can be brought on by this bias and result in worse-than-ideal investing outcomes.

The actions of many individual investors during the 2008 financial crisis, when they panicked and sold their investments at the bottom of the market to prevent additional losses, locking in substantial losses in the process, serve as a meaningful illustration of the loss aversion bias. (Barber and Odean, 2008).

Confirmation Bias:

Confirmation bias is the propensity of people to seek out and interpret information that supports their preexisting opinions while disregarding or dismissing evidence that deviates from those beliefs.

According to Grinblatt and Keloharju (2000), the “underweightment” of information that contradicts investors preconceived notions and “overweightment” of information that supports those notions results in the self-reinforcing cycle where investors make potentially poor investing judgments because they increasingly grow more certain of their beliefs as they come across confirming facts.

The bursting of the dot-com bubble is again an exemplary event where investors showed confirmatory biases. As investors were ignoring the excessive valuations and the lack of earnings, they were paying attention to data that supported their positive viewpoint on the transforming power the internet would have on the technology industry. Many of these investors experienced large losses when the technology stocks fell in 2000.

The Enron corporation can be another example of confirmation bias in action. The big energy company collapsed due to accounting fraud in the early 200s. Before its disastrous collapse, many investors held positive beliefs in the future success of Enron, reinforced by optimistic news coverage, their high stock price, and the comments of analysts. This confirmation bias obstructed many investors from critically evaluating the company's financial statements, resulting in incredible losses with the bankruptcy of the company.

Reconciling Modern Portfolio Theory & Behavioral Financing: Behavioral Portfolio Theory:

Having established the stark contrast between Modern Portfolio Theory based on the assumption of efficient markets and Behavioral finance which challenges this theory. Following these opposing views, economists Hersh Shefrin and Meir Statman released a groundbreaking paper in 2000 which reconciled these differences through the introduction of something we now know of as Behavioral Portfolio Theory. Behavioral Portfolio theory varies from Traditional finance in the way it minimizes the risk and return tolerance of investors.

Traditional Finance:
Traditional finance seeks to reduce risk by incorporating diverse assets in an investor's portfolio that are mean-variance efficient. Mean-variance analysis is a method of calculating the risk associated with an asset by comparing its variance against its expected return. According to traditional finance, investors are concerned with the expected return and variance of their portfolio. Traditional finance basically groups the aims of all investors to be one and the same that is profit maximization. However, as proposed by the Behavioral Finance theory this is not the case. Investors have varied aims and construct their portfolios accordingly. Shefrin and Statman developed the BPT based on two key theories, the Security Potential/Aspiration (SP/A) theory by Lopez and the Prospect Theory by Kahneman and Tversky.

SP/A Theory:
The SP/A Theory states that investors decisions involve emotional aspects such as the relative "security" or safety from loss, the "potential" of earning higher levels of wealth and the "aspiration" of achieving their personal aims.

Prospect Theory:
The prospect theory states that several misconceptions influence the decision-making process. Here instead of using probabilities as weights to evaluate the utility function of investors, we use the respective decision weights which is dependent on the individual loss and gains of the assets that make up the portfolio rather than the overall wealth. Essentially stating that the overall value of a portfolio to an investor is heavily affected by the certainty affect (the psychological behavior of individuals to choose a certain event over a probable event) and the isolation effect (the notion that people have different ways to reach to the same outcome, in this case different asset combinations to reach the same profit).

The Behavioral Portfolio Theory:
The BPT in its essence states that investors do not consider their portfolio as a single set. Instead, they consider their portfolio as multiple sub portfolios, each associated to a certain goal. These are layered in the form of a pyramid. One layer might be a "downside protection" layer, designed to protect investors from being poor. Another might be an "upside potential" layer, designed to give investors a chance at being rich (figure 1).

BPT is usually proposed in two basic versions, usually differentiated by the level of correlation between the assets that make up the portfolio:

- Single Mental Account (BPT-SA)
- Multiple Mental Account (BPT-MA)

In the BPT-SA model investors consider the portfolio as a whole or as a single mental account. In this case their decisions are heavily influenced by the correlations between the assets that make up the portfolio, so when there is no short selling, and the distributions of returns are normal, BPT-SA portfolios may be at the efficient frontier. The BPT-SA model can be confused to be similar to the mean variance model which considers the portfolio as a whole as well. However, in the mean variance model we consider the attitude of investors to risk being uniform that is either risk averse or risk loving, and these preferences are seen in their utility curves. In contrast, behavioral investors have many attitudes toward risk, layer by layer. In BPT we have different scenarios where investors may be risk loving as long as it involves a little sum of money but may be risk averse when it involves a greater proportion of their wealth. Thus, the concept of risk itself is diluted rather than absolute in the case of BPT.

In contrast to the BPT-SA, the BPT-MA model, assets have little correlation and hence are treated as if they were independent portfolios. BPT-MA investors segregate their portfolios into mental accounts and overlook covariance among mental accounts. For example, BPT-MA investors might place foreign stocks in one mental account and domestic stocks in another. They might consider foreign stocks highly risky because they overlook the effect that the covariance between foreign and domestic stocks exerts on the risk of the portfolio when viewed as an integrated single account. BPT-MA portfolios resemble layered pyramids where each level is associated with their aspirations, in general a low aspiration level(downside protection layer) is aimed at avoiding poverty and a high aspiration layer (upside protection layer) is designed for a shot riches.

Consider an investor who is pulled equally strongly by the desire for downside protection and the desire for upside potential, so he divides his $2 into two equal parts, $1 for the downside protection layer and $1 for the upside potential layer. Imagine that the aspiration level of that investor for the downside protection layer is $1, a level that is low relative to the $1.20 aspiration level for the upside potential layer. Imagine that there were two securities, X and Y, both of which have normally distributed returns. X has an expected return of 16 percent with a standard deviation of 20 percent, while Y has an expected return of 10 percent with a standard deviation of 15 percent. The correlation between the returns of X and Y is zero.
Now we contemplate behavioral efficient frontier for the downside protection layer. A portfolio consisting entirely of Y offers an expected wealth of $1.10 along with a 25.2 percent probability that the aspiration level will be missed. Portfolio Y is not on the efficient frontier since it is dominated by portfolio Z. Portfolio Z, combining $0.50 of X with $0.50 of Y, has the lowest probability of missing the aspiration level, 14.9 percent, along with higher expected wealth,

$1.13. Figure 2 shows the efficient frontier for the downside protection layer, extending from Z to X. Next, consider the efficient frontier for the upside potential layer. Figure 3 shows that the frontier consists of a concentrated bet on security X. Any other portfolio is inferior since it provides a lower expected wealth with a higher probability of missing the aspiration level.

Conclusion:

In conclusion we can say that the very concept of gains and losses that were though of being absolute and very mathematical in their derivations as suggested by through the findings of Modern Portfolio Theory are being challenged and the very field of finance is becoming increasingly relative which has given rise to new theories that discuss the concept of optimizing profit with these new assumptions. This has brought importance to the concept of market inefficiencies that drive these theories. As we move further away from perfect markets, knowing these inefficiencies and incorporating them in decision making is the basis of Behavioral Finance. Moreover, every investor has a unique investment style and thus we cannot use a "one-size-fits-all" approach to derive formulas for portfolio maximization. The idea isn't to completely disregard either, rather it is to alter the formulas and theories brought forward by the Modern Portfolio Theory in the mid 1900's with the ideas put forward in the early 200s about Behavioral Finance, which is exactly what Shefrin and Statmen did with their paper in 2000, which enables us to take into consideration these inefficiencies to the maximum level possible.

Tables & Graphs:

Figure 1:
Portfolios as layered pyramids.
Source: Putnam Investments (2003).

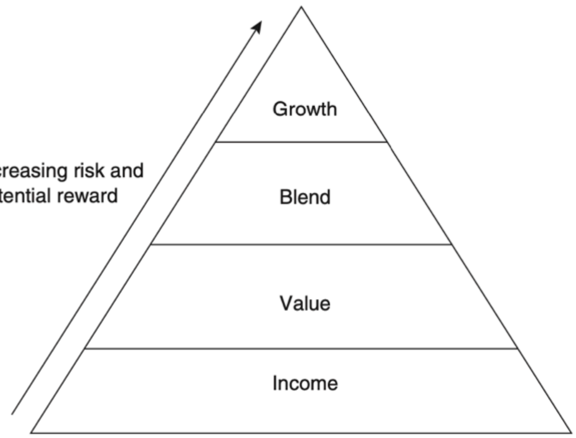


Figure 4-5. Portfolios as layered pyramids.
Source: Putnam Investments (2003).

Figure 2:
Behavioral efficient frontier for an investor with $1.00 in the downside protection layer and a $1.00 aspiration level.
Source: Shefrin and Statman (2000).

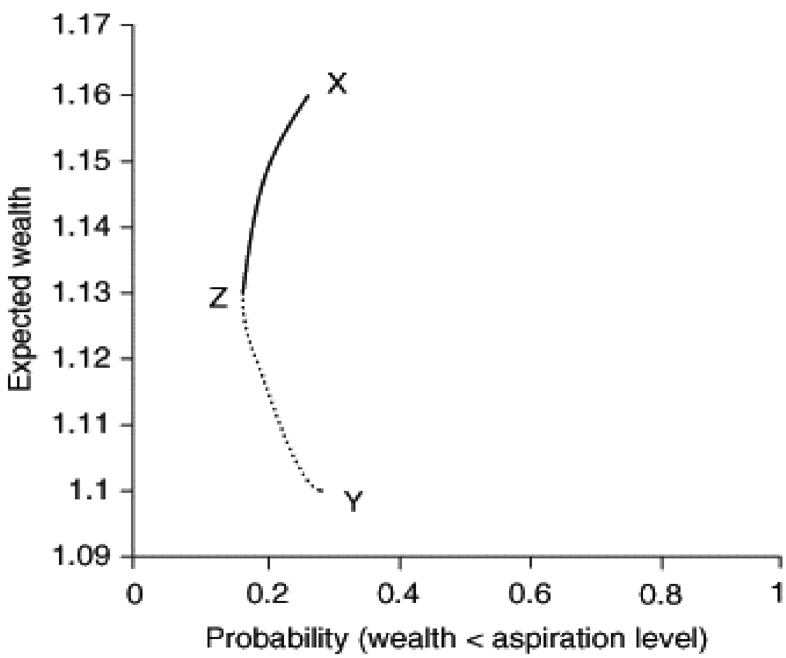
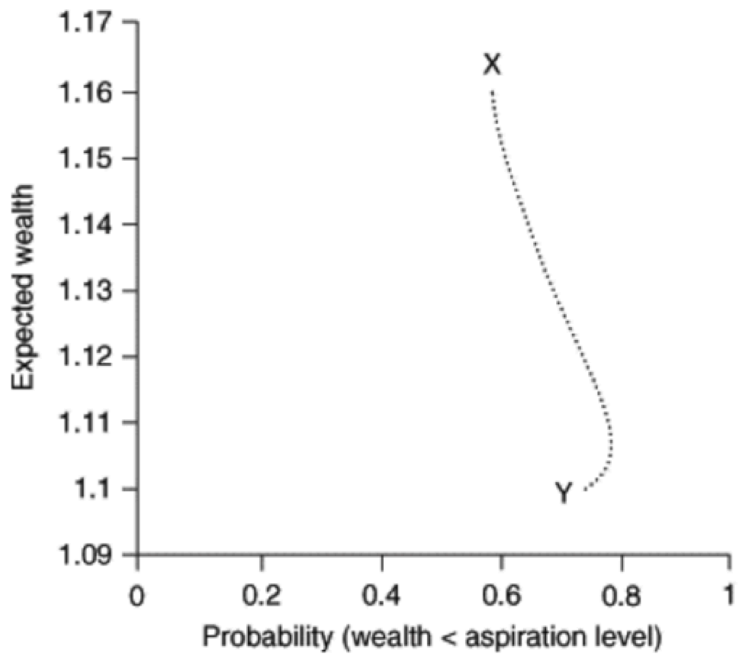


Figure 3:
Behavioral efficient frontier for an investor with $1.00 in the upside potential layer and a $1.20 aspiration level.
Source: Shefrin and Statman (2000).



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